INTERNATIONAL BACCALAUREATE

Mathematics: applications and interpretation

MAI

EXERCISES [MAI 1.7]

INFINITE SERIES

Compiled by Christos Nikolaidis

A. Paper 1 questions (SHORT)

1. [Maximum mark: 8]

Consider the geometric sequence 10, 5, 2.5, 1.25, …

(a) Express the general term u_n in terms of n. [1]

(b) Find the first term which is smaller than $10^{-3} = 0.001$. [3]

(c) Find the sum of the first 20 terms correct to **6 decimal places**. [2]

(d) Find the sum of the infinite series. [2]

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2. [Maximum mark: 6]

Find the sum of each of the following infinite geometric series

(i) $1+\frac{2}{5}+\frac{4}{25}+\frac{8}{125}+\cdots$ (ii) $1-\frac{2}{5}+\frac{4}{25}-\frac{8}{125}+\cdots$

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3. [Maximum mark: 6]

Calculate the following sums by using the appropriate formulas

(i) $\sum_{k=1}^{6} 4^k$ (ii) $\sum_{k=1}^{6} (0.25)^k$ (correct to **6 dp**) (iii) $\sum_{k=1}^{+\infty} (0.25)^k$

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4. [Maximum mark: 5]

Consider the infinite geometric sequence $3, 3(0.9), 3(0.9)^2, 3(0.9)^3, \dots$.

(a) Write down the 10th term of the sequence. Do not simplify your answer. [1]

(b) Find the sum of the infinite sequence. [4]

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5. [Maximum mark: 6]

Consider the infinite geometric sequence $25, 5, 1, 0.2, \dots$.

(a) Find the common ratio. [1]

(b) Find

(i) the 10th term; (ii) an expression for the n^{th} term. [3]

(c) Find the sum of the infinite sequence. [2]

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6. [Maximum mark: 6]

Consider the infinite geometric series $405 + 270 + 180 + ...$

(a) For this series, find the common ratio, giving your answer as a fraction in its simplest form. [2]

(b) Find the fifteenth term of this series. [2]

(c) Find the **exact** value of the sum of the infinite series. [2]

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7. [Maximum mark: 4]

Consider the infinite geometric sequence

$$3000, -1800, 1080, -648, \ldots .$$

Find the **exact** sum of the infinite sequence.

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8. [Maximum mark: 5]

The first three terms of an infinite geometric sequence are 32, 16 and 8.

(a) Write down the value of r. [1]

(b) Find u_6. [2]

(c) Find the sum to infinity of this sequence. [2]

9. [Maximum mark: 4]

Find the sum of the infinite geometric sequence 27, −9, 3, −1, ...

10. [Maximum mark: 4]

Find the sum of the infinite geometric series

$$\frac{2}{3}-\frac{4}{9}+\frac{8}{27}-\frac{16}{81}+\ldots$$

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11. [Maximum mark: 6]

The first term of an infinite geometric sequence is 18, while the third term is 8. There are two possible sequences. Find the sum of each sequence.

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12. [Maximum mark: 4]

Find the sum to infinity of the geometric series

$$-12+8-\frac{16}{3}+....$$

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13. [Maximum mark: 6]

The first and fourth terms of a geometric series are 18 and $-\frac{2}{3}$ respectively. Find

(a) the sum of the first n terms of the series; [4]

(b) the sum to infinity of the series. [2]

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14. [Maximum mark: 6]

An infinite geometric sequence has first term u_1 and common ratio r. Find the values of u_1 and of r given that $S_3 = 35$ and $S_\infty = 40$.

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15. [Maximum mark: 6]

A geometric sequence has all positive terms. The sum of the first two terms is 15 and the sum to infinity is 27. Find the value of

(i) the common ratio; (ii) the first term.

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16. [Maximum mark: 6]

The sum to infinity of a geometric series is 32. The sum of the first four terms is 30 and all the terms are positive. Find the difference between the sum to infinity and the sum of the first eight terms.

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17. [Maximum mark: 4]

Let $\sum_{x=0}^{\infty} k\left(\frac{2}{3}\right)^{x} = 1$. Find the value of k.

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18. [Maximum mark: 7]

Consider the sequence $x-3,\ x+1,\ 2x+8,\ \ldots$

(a) Find the values of x for which the sequence is geometric. [3]

(b) State the value of x, for which the series has a finite sum (converges). [2]

(c) For this value of x, found in (b) find the sum of the infinite series. [2]

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19. [Maximum mark: 6]

Consider the infinite geometric series $1+\left(\frac{2x}{3}\right)+\left(\frac{2x}{3}\right)^2+\left(\frac{2x}{3}\right)^3+.....$

(a) For what values of x does the series converge? [3]

(b) Find the sum of the series if $x=1.2$. [3]

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20. [Maximum mark: 6]

An infinite geometric series is given by $\sum_{k=1}^{\infty} 2(4-3x)^k$

(a) Find the values of x for which the series has a finite sum. [3]

(b) Find the sum of the series when $x=1.2$. [3]

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B. Paper 2 questions (LONG)

21. [Maximum mark: 11]

The diagrams below show the first four squares in a sequence of squares which are subdivided in half. The area of the shaded square A is $\frac{1}{4}$.

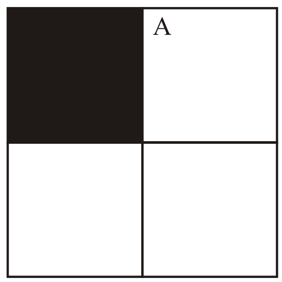


Diagram 1

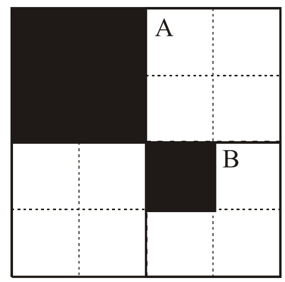


Diagram 2

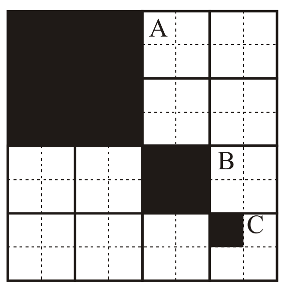


Diagram 3

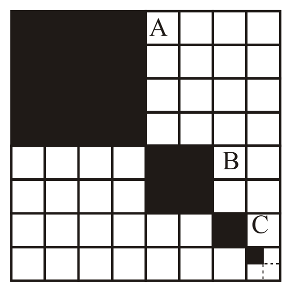


Diagram 4

(a) (i) Find the area of square B and of square C.

(ii) Show that the areas of squares A, B and C are in geometric progression.

(iii) Write down the common ratio of the progression. [5]

(b) (i) Find the **total** area shaded in diagram 2.

(ii) Find the **total** area shaded in the 8^{th} diagram of this sequence.

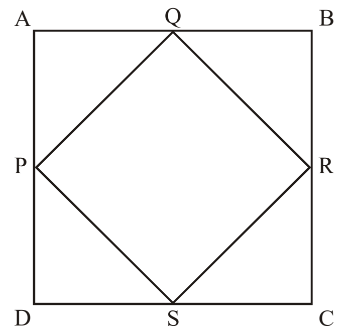
Give your answer correct to six significant figures. [4]

(c) The dividing and shading process illustrated is continued indefinitely.

Find the total area shaded. [2]

22. [Maximum mark: 10]

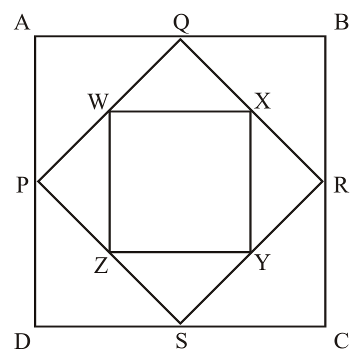
The diagram shows a square ABCD of side 4 cm. The midpoints P, Q, R, S of the sides are joined to form a **second** square.



(a) (i) Show that PQ = $2\sqrt{2}$ cm.

(ii) Find the area of PQRS. [3]

The midpoints W, X, Y, Z of the sides of PQRS are now joined to form a **third** square as shown.



(b) (i) Write down the area of the **third** square, WXYZ.

(ii) Show that the areas of ABCD, PQRS, and WXYZ form a geometric sequence. Find the common ratio of this sequence. [3]

The process of forming smaller and smaller squares (by joining the midpoints) is **continued indefinitely.**

(c) (i) Find the area of the 11th square.

(ii) Calculate the sum of the areas of **all** the squares. [4]

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23. [Maximum mark: 14]

(a) Find the three roots of the equation $4p^3 + 4p^2 - 15p - 8 = 0$. [3]

A geometric sequence has first term 4 and p as its common ratio.

An arithmetic sequence has first term 4 and $5p$ as its common difference.

(b) Write down, in terms of p, the first four terms of the geometric sequence [2]

(c) Write down, in terms of p, the first four terms of the arithmetic sequence [2]

(d) If the sum of the third and fourth terms of the geometric sequence is equal to the sum of the first and fourth terms of the arithmetic sequence, find the three possible values of p. [3]

For some value of p, among those found in (d), the sum to infinity of the terms of the geometric sequence exists

(e) Write down this value of p and find the sum to infinity. [4]

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For the following 2 questions you have to know the theory of exponents & logarithms.

24. [Maximum mark: 13]

Consider the series

$$1+e^{x}+e^{2x}+e^{3x}+\cdots$$

(a) Given that $x=1$, find the sum of the **first ten** terms. [3]

(b) Given that $x=-1$, find the sum of the **first ten** terms. [2]

(c) Given that $x=-1$, explain why the **infinite series** converges and find its sum in the form $\frac{e}{e+a}$ where a is a constant to be determined. [3]

(d) Given that the sum of the infinite series is equal to 2, find the value of x,

(i) in the **exact** form $\ln a$ (ii) correct to 3 s.f. [5]

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25. [Maximum mark: 15]

(a) Explain why the series $\log x + \log x^2 + \log x^3 + \cdots$ is **arithmetic**. [3]

(b) Find the sum of the first **ten** terms of the series $\log 3 + \log 3^2 + \log 3^3 + \cdots$ [3]

(c) Explain why the series $\log x + (\log x)^2 + (\log x)^3 + \cdots$ is **geometric**. [2]

(d) Find the sum of the first **ten** terms of the series $\log 3 + (\log 3)^2 + (\log 3)^3 + \cdots$ [3]

Consider now the **infinite** series

$$1 + \log 3 + (\log 3)^2 + (\log 3)^3 + \cdots$$

(e) Explain why the series converges and find its sum. [4]

Give all numerical answers in 3 s.f.

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